Exam
Mechanics & Relativity 2014–2015 (part Classical Mechanics)
January 23, 2015, 9:00–12:00hr

INSTRUCTIONS

- This exam comprises 5 problems. Write your solution of *each problem on a separate sheet.*
- The first is a set of four conceptual multiple-choice questions, for which only the answer matters not your arguments. The answers to problems 2 through 5 require clear arguments and derivations, all written in a well-readable manner.
- The number of points for every subquestion are indicated inside a box in the margin. The total number of points per problem is

Problem	# of points
1	4
2	3
3	5
4	3
5	1

and the grade is computed as (total # points + 1) /1.7. Not counting any homework bonus, a minimum of 9 points is needed to pass the exam.

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PROBLEM 1 *Below are four multiple-choice questions: only the answer (capital letter A thru E) matters, not your arguments.*

a. A boy throws a steel ball straight up. Consider the motion of the ball only after it has left the boy's hand but before it touches the ground, and assume that forces exerted by the air are negligible. For these conditions, the force(s) acting on the ball is (are):

A) a downward force of gravity along with a steadily decreasing upward force.

B) a steadily decreasing upward force from the moment it leaves the boys hand until it reaches its highest point; on the way down there is a steadily increasing downward force of gravity as the object gets closer to the earth.

C) an almost constant downward force of gravity along with an upward force that steadily decreases until the ball reaches its highest point; on the way down there is only a constant downward force of gravity.

D) an almost constant downward force of gravity only.

E) none of the above. The ball falls back to ground because of its natural tendency to rest on the surface of the earth. 1

b. The figure below shows a boy swinging on a rope, starting at a point higher than P. Consider the following distinct forces:

1. a downward force of gravity.

2. a force exerted by the rope pointing from P to O.

3. a force in the direction of the boys motion.

4. a force pointing from O to P.

Which of the above forces is (are) acting *on* the boy when he is at position P?

A) 1 only.

B) 1 and 2.

C) 1 and 3. 1

D) 1,2 and 3.

E) 1, 3 and 4.

c. A skier gliding downhill at a steady speed is subjected to the following forces:

1. a downward force of gravity

2. friction on the skies

3. air resistance

Which of the above forces is (are) responsible for the observation that heavier people are faster?

A) 1 only.

B) 1 and 2.

C) 1 and 3.

D) 2 and 3.

E) 1, 2 and 3

1

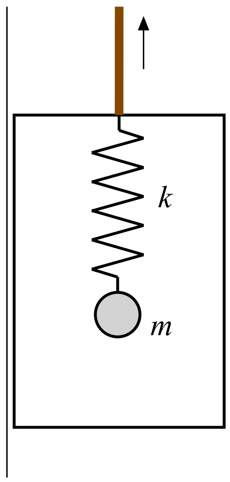
d.

A mass–spring system is suspended from the ceiling of an elevator. When the elevator accelerates upwards, the frequency of this harmonic oscillator,

A) remains the same

B) decreases

C) increases



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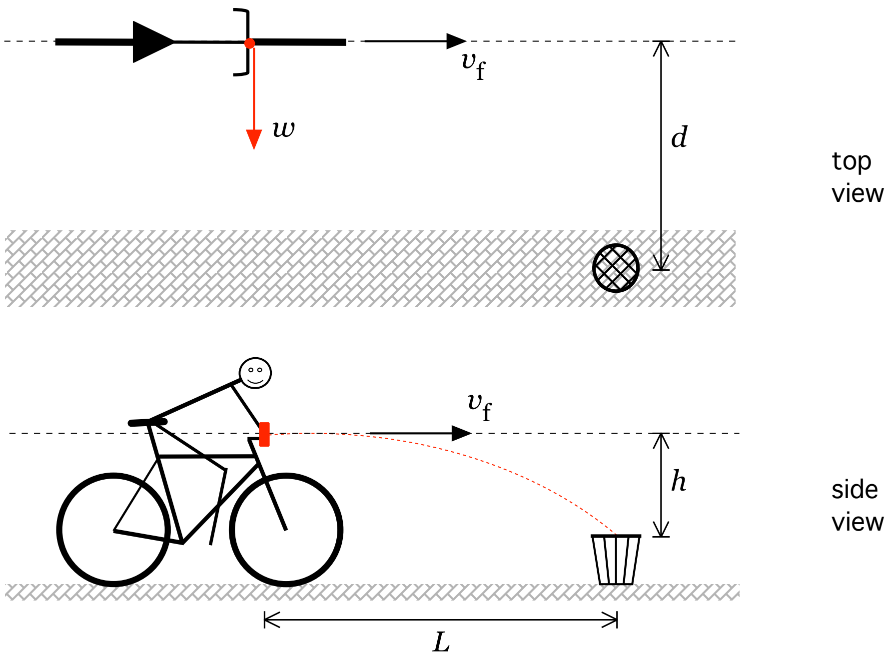
The answers to problems 2 through 5 require clear arguments and derivations, all written in a neat and well-readable manner and supported by clear drawings if necessary.

PROBLEM 2 You are riding your bicycle at a constant velocity v_f, holding an empty beverage can that you want to get rid of. By the side of the road ahead, you notice a bin and you decide to toss the can by throwing it to the side. When should you do so, and with what velocity in order that the can falls into the bin?



With the following simplifying assumptions:

- you keep riding parallel to the curb, and the distance between the bin and your route is d;
- you throw the can in the horizontal direction with a velocity w perpendicular to your route;
- the dimensions of the can and the bin can be neglected;
- there is no air resistance;
- the can is launched from the mid-plane of the bike;
- the height difference between the initial position of the can and the top of the bin is h,

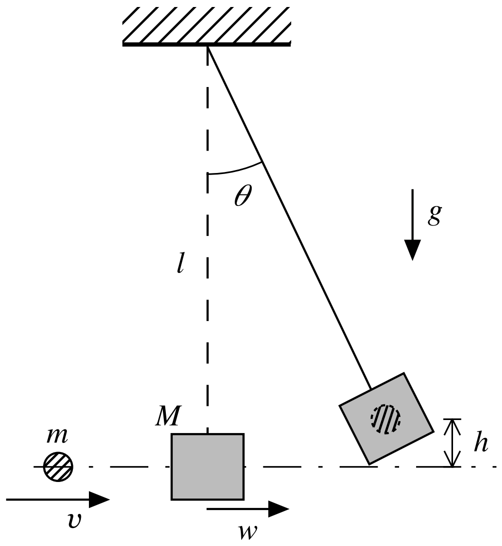
we can now analyse this problem.

a. Suppose we would already know the distance L at which you should toss the can, with which sideways velocity w should you do so? [1]

b. Given the initial velocity components v_f and w in the horizontal plane, how long does it take for the can to fall *down* into the bin? [1]

c. At what distance L should you launch the can with speed w so that it ends up in the bin? [1]

PROBLEM 3 In old times, a so-called ballistic pendulum was used to measure the speed of bullets. For this purpose, a box filled with sand was suspended from the ceiling with a rope of length l. The bullet was shot in horizontal direction into the box and caught there by the sand. Due to the injected momentum, the box will swing away and kinetic energy is transformed into potential energy. By measuring the maximum potential energy, one can deduce the initial velocity of the bullet.
In the model below, the mass of the rope can be neglected. The mass of the bullet is m, its velocity is v and the mass of the sand-filled box is M.



a. Show that the velocity w of the bullet immediately after it has been caught by the box is given by

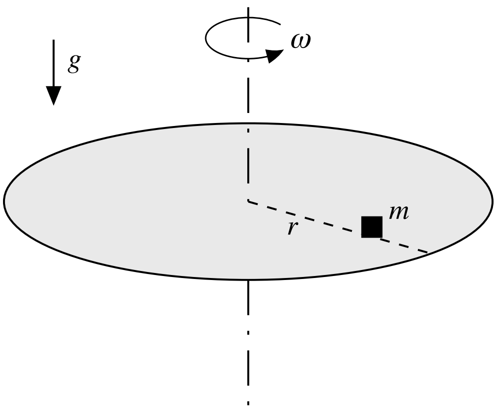
$$w = \frac{m}{m+M} v$$

1

b. Which conservation law describes how high box and bullet can swing upwards? Given the maximum deflection of the pendulum, θ, determine the bullet's initial velocity v. 3

c. If the box would not be stopped, it will swing back and forth. With what frequency? 1

PROBLEM 4 A small beetle of mass m is standing on a disc that rotates about a vertical axis at constant angular frequency ω. The coefficient of friction between beetle and disc is μ.



a. Which forces does the beetle observe (in his co-moving frame of reference) when it is positioned at a distance r from the axis? Specify direction and magnitude of all forces.

b. Determine the maximum velocity of the disc for which the beetle is able to remain at rest on the disc.

c. Describe the forces when he is not at rest but moves radially towards the center of the disc. Draw these forces in a free-body diagram of the beetle.

3×1

PROBLEM 5 Bob and Ann disagree about the sign in the expression for the potential energy of a spring with linear stiffness k. Bob argues as follows:

- In order to stretch the spring over a distance x, I have to apply a force of magnitude kx in the direction of x;
- Substitution of this in the definition of potential energy $V(x)$ then yields

$$
\begin{aligned}
V(x) &= -\int_0^x F(x')dx' \\
&= -k\int_0^x x'dx' = \\
&= -\frac{1}{2}kx^2 \,.
\end{aligned}
$$

However, according to Ann, the potential energy of a spring is given by $V = +\frac{1}{2}kx^2$.
Question: who is right and, more importantly, why? 1

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Answers
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Problem 1

a. D

b. B

c. C

d. A

Problem 2

a. $w = v_\mathrm{f} d/L$.

b. $t = \sqrt{2h/g}$.

c. $L = tv_\mathrm{f} = \sqrt{2h/g}v_\mathrm{f}$.

Problem 3

a. Conservation of momentum: $mv = (m + M)w$.

b. Conservation of energy after impact:

$$\frac{1}{2}(m + M)w^2 = (m + M)gh \Longrightarrow h = \frac{w^2}{2g}$$

With the answer from (a):

$$v = \frac{m + M}{m}\sqrt{2gh} \approx \frac{m + M}{m}\theta\sqrt{gl}$$

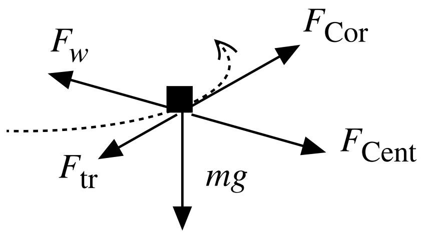
c. $\omega = \sqrt{g/l}$ or $f = \frac{1}{2\pi}\sqrt{g/l}$.

Problem 4

a. (i) gravity mg; (ii) friction force $F_w < \mu mg$; (iii) centrifugal force $F_\mathrm{Cent} = m\omega^2 r$.

b. Maximum speed of rotation is $\omega = \sqrt{\mu g/r}$.

c. The Coriolis force with magnitude $F_\mathrm{Cor} = 2m\omega v$, pointing in the direction of rotation (in addition to the forces in (a)).



NB: ignore the force F_{tr} in the diagram above; this is a leftover of a previous more elaborate version of this problem.

Problem 5
Ann is right because the force to be used in $V(x) = -\int_0^x F(x')dx'$ is the force exerted by the spring *on* the particle. Also, the energy in a spring is always positive.